

UM-TH-95-08

March 1995

(revised April 1995)

hep-ph/9503445

Low-energy supersymmetry with D-term contributions to scalar masses

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ABSTRACT: We investigate how the predictions of the Minimal Supersymmetric Standard Model are modified by D-term contributions to soft scalar masses, which arise whenever the rank of the gauge group at very high energies is greater than four. We give a parameterization of the most general such contributions that can occur when the unbroken gauge symmetry is an arbitrary subgroup of E_6 , and show how the D-term contributions leave their imprint on physics at ordinary energies. We impose experimental constraints on the resulting parameter space and discuss some features of the resulting supersymmetric spectrum which differ from the predictions obtained with universal boundary conditions on scalar masses near the Planck scale. These include relations between squark and slepton masses; the behavior of $\sin^2(\beta - \alpha)$ (which determines the production cross-section for the lightest Higgs scalar boson at an e^+e^- collider) and the mass of the pseudoscalar Higgs bosons; R_b [the ratio $\Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$]; and mass differences between charginos and neutralinos.

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1. Introduction

The mass of the Higgs scalar boson in the standard model is subject to quadratically divergent radiative corrections of order $\delta m_H^2 \sim \Lambda^2$, where Λ is an ultraviolet cutoff mass. This poses a naturalness problem for Λ much larger than the electroweak scale. Low-energy supersymmetry (SUSY) [1] presents a beautiful solution to this problem, because all quadratic divergences cancel order-by-order in perturbation theory. The cancellation still works if SUSY is softly broken by scalar mass terms, scalar trilinear couplings, and gaugino mass terms. These are exactly the parameters which determine the masses of the superpartners of standard model particles; therefore, in the minimal supersymmetric standard model (MSSM), the superpartners must have masses which do not greatly exceed the TeV range in order to maintain the stability of the electroweak scale. Moreover, we already know that the soft SUSY-breaking terms in the MSSM are far from arbitrary in their flavor and phase structure. Otherwise, large flavor changing neutral currents could be expected to manifest themselves in processes such as $K - \bar{K}$ mixing, $b \rightarrow s\gamma$, and $\mu \rightarrow e\gamma$. Arbitrary complex phases would give rise to a CP-violating electric dipole moment for the neutron in violation of experimental bounds. Thus there is strong circumstantial evidence in favor of some organizing principle governing the soft SUSY-breaking terms.

Supergravity models [2] do provide just such an organizing principle for the soft terms, if one makes the assumption that gravity is flavor blind. SUSY is presumed to be broken at a scale $M_I \sim 10^{10}$ GeV in a “hidden” sector of particles which have only gravitational interactions with the “visible” sector of particles familiar to us. The SUSY breaking is transmitted to the visible sector by gravitational effects, so that the soft SUSY-breaking terms in the visible sector are characterized by a mass scale $m_{\text{SUSY}} = M_I^2/M_{\text{Planck}} \sim M_Z$ and are universal, independent of the unknown physics of the hidden sector. This mechanism implies a simple form for the soft breaking Lagrangian at the Planck scale. The following terms arise:

- A common (mass)² (denoted m_0^2) for all scalars in the theory.
- Scalar trilinear couplings which are given by the corresponding Yukawa couplings in the

superpotential multiplied by a common mass parameter A_0 .

- Scalar (mass)² terms given by the corresponding mass in the superpotential multiplied by a common mass parameter B_0 .
- A common mass $m_{1/2}$ for the gauginos.

The mass parameters m_0 , A_0 , B_0 , and $m_{1/2}$ are all of order m_{SUSY} . The assumption of universality may have to be modified in non-minimal supergravity models [3] or in superstring models [4]; it is clear in any case that while they are sufficient to avoid flavor-changing neutral currents, they are certainly not necessary. The origins and consequences of scalar mass non-universalities which are safe for flavor-changing neutral currents have been discussed for example in [5-16].

In the MSSM, the superpotential is given by

$$W = uY_uQH_u + dY_dQH_d + eY_eLH_d + \mu H_uH_d$$

where Q, L (u, d, e) are chiral superfields for the $SU(2)_L$ doublet (singlet) quarks and leptons; H_u, H_d are the Higgs doublet chiral superfields with weak hypercharge $+1/2, -1/2$; and Y_u, Y_d, Y_e are 3×3 Yukawa matrices. The simple supergravity assumptions above tell us that the soft SUSY-breaking Lagrangian at M_{Planck} is

$$\begin{aligned} L_{\text{soft}} = & -m_0^2 \left[|H_u|^2 + |H_d|^2 + \sum_{\text{families}} (|Q|^2 + |u|^2 + |d|^2 + |L|^2 + |e|^2) \right] \\ & - A_0(uY_uQH_u + dY_dQH_d + eY_eLH_d) + \text{H.c.} \\ & - B_0\mu H_uH_d + \text{H.c.} \\ & - \frac{1}{2}m_{1/2}(\tilde{g}\tilde{g} + \tilde{W}\tilde{W} + \tilde{B}\tilde{B}) + \text{H.c.} \end{aligned} \tag{1.1}$$

(Here we use the same symbol for the scalar fields as for the chiral superfields, and $\tilde{g}, \tilde{W}, \tilde{B}$ are the gauginos for $SU(3)_C, SU(2)_L, U(1)_Y$ respectively.) This parametrization serves as a set of boundary conditions on the model. The couplings so specified must be run down to ordinary energies using the renormalization group (RG) equations. This has been done by many groups; recent examples include [17-24].

Of course, it is quite unlikely that there is no new physics between the TeV scale and the Planck scale. A much-heralded hint of this is the apparent unification of gauge couplings at $M_U \approx 2 \times 10^{16}$ GeV with SUSY thresholds near the electroweak scale[25]. This suggests a SUSY grand unified theory (GUT) or superstring model, both of which in turn imply a variety of new chiral superfields and new gauge interactions at least two orders of magnitude below the Planck scale at which the boundary conditions (1.1) should be applied. The existence of this physics should be taken into account by running the gauge couplings of the theory from M_{Planck} down to the weak scale, integrating out non-MSSM fields along the way as they acquire masses. Unfortunately, it is very difficult to anticipate with any confidence the nature of the physics between M_U and M_{Planck} . For this reason, most studies have used the approximation of applying the boundary conditions (1.1) at M_U rather than M_{Planck} . Some recent papers[10,12,14] have taken into account the effects of RG running between M_U and M_{Planck} , which can be large e.g. in certain GUT models.

In this paper we will be mainly concerned with the D-term contributions [5] to soft scalar masses which should arise in any model with a gauge group of rank > 4 which breaks down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Some of the phenomenological implications of D-term contributions have recently been explored in [6,7,9,10,11,15]. We will consider here the case that the underlying gauge group is an arbitrary rank 5 or 6 subgroup of E_6 . This encompasses a very wide range of special cases, including $SO(10)$ SUSY GUTs and string-inspired models including e.g. “flipped” $SU(5) \times U(1)$ and $SU(3)_C \times SU(3)_L \times SU(3)_R$. The breaking of the additional $U(1)$ factors embedded within such groups leads to specific patterns in the soft scalar masses which may be thought of as non-universal corrections to the universal boundary conditions (1.1), even though they typically arise at a much lower scale. These non-universal corrections to the scalar masses do not imply additional contributions to flavor-changing neutral current processes, since the contributions are the same for each set of squarks and sleptons with the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers, e.g. $(\tilde{d}_L, \tilde{s}_L, \tilde{b}_L)$. The D-terms allow one to reach certain regions of parameter space which cannot be accessed using the universal boundary conditions, sometimes with interesting implications for phenomenology below the TeV scale. For example,

much smaller values of μ can be obtained for fixed values of the other parameters if the D-term contributions have the appropriate sign and magnitude. The Higgs pseudoscalar mass might be lower than predicted in the universal case, and the lightest neutral Higgs eigenstate may have couplings which differ significantly from those of a standard model Higgs scalar of the same mass. The D-terms may also leave a dramatic imprint on sum rule relations among the squark and slepton masses. Such effects may eventually help to distinguish between various scenarios for gauge symmetry breaking at very high energies.

This paper is organized as follows. In section 2 we will review the origin of D-term contributions to soft scalar masses, and introduce a parametrization of the most general such contributions arising in the MSSM from arbitrary subgroups of E_6 . Using the RG equations we will show how the D-term contributions at very high energies leave their imprint on TeV scale physics. In section 3 we will discuss some of the salient phenomenological consequences of the D-term-induced non-universality, taking into account direct and indirect experimental constraints. In section 4 we will make some concluding remarks.

2. D-term contributions to soft scalar masses

In general, D-term contributions to the scalar masses will arise whenever a gauge symmetry is spontaneously broken with reduction of rank [5]. To understand this, consider a toy model with a gauge group containing a product of abelian factors $\prod_I U(1)_I$. We take the $U(1)_I$ gauge couplings and charges to be normalized so that the $U(1)_I$ gauge supermultiplets have canonical kinetic terms. This determines, up to orthogonal rotations on the index I , a preferred basis for the $U(1)_I$ charges which is *not* invariant under general invertible linear transformations. The gauge group is to be spontaneously broken by VEVs for the scalar components of chiral superfields Φ and $\bar{\Phi}$ with charges $Q_{I\Phi}$ and $-Q_{I\Phi}$ respectively under $U(1)_I$. For illustrative purposes, we may for example suppose that the spontaneous symmetry breaking is accomplished by taking the superpotential to be

$$W = \frac{1}{nM^{2n-3}}(\Phi\bar{\Phi})^n \quad (n \geq 2)$$

with M a mass parameter of order the Planck or string scale. The supersymmetric part

of the scalar potential including D-terms is given by

$$V_{\text{SUSY}} = \frac{1}{M^{4n-6}} (|\Phi|^2 + |\bar{\Phi}|^2) |\Phi \bar{\Phi}|^{2n-2} + \sum_I \frac{g_I^2}{2} \left(Q_{I\Phi} (|\Phi|^2 - |\bar{\Phi}|^2) + \sum_a Q_{Ia} |\varphi_a|^2 \right)^2 \quad (2.1)$$

where the additional fields φ_a play the role of the scalar fields of the MSSM. In addition there are soft SUSY-breaking terms which include

$$V_{\text{soft}} = m^2 |\Phi|^2 + \bar{m}^2 |\bar{\Phi}|^2. \quad (2.2)$$

The full scalar potential has a non-trivial VEV in a nearly D-flat direction:

$$\langle \Phi \rangle^2 \approx \langle \bar{\Phi} \rangle^2 \approx \left[\frac{-(m^2 + \bar{m}^2) M^{4n-6}}{(4n-2)} \right]^{1/(2n-2)} \quad (2.3)$$

provided that $m^2 + \bar{m}^2 < 0$ at the scale $\langle \Phi \rangle$. This can be easily achieved if for example Φ has large Yukawa couplings to some other superfields, driving the running m^2 negative in the infrared. The deviation from D-flatness is given by

$$\langle \Phi \rangle^2 - \langle \bar{\Phi} \rangle^2 \approx \frac{1}{2} (\bar{m}^2 - m^2) / \sum_I g_I^2 Q_{I\Phi}^2. \quad (2.4)$$

After integrating out the fields Φ and $\bar{\Phi}$, one obtains corrections to the soft scalar masses of the remaining fields φ_a :

$$\Delta m_a^2 = \sum_I Q_{Ia} d_I; \quad d_I = \frac{1}{2} (\bar{m}^2 - m^2) g_I^2 Q_{I\Phi} / \sum_J g_J^2 Q_{J\Phi}^2. \quad (2.5)$$

From (2.5) we see that the D-term contributions do not depend on the precise form of the superpotential (2.1), and only depend on the soft masses m^2 , \bar{m}^2 and the charges under the $U(1)_I$ gauge groups which participate in the breaking. The VEVs (2.3) reduce the rank of the gauge group by 1, with the surviving $N-1$ $U(1)$'s having charges which are linear combinations of the N original $U(1)_I$ charges. The contributions to the $(\text{mass})^2$ of the surviving scalar fields are just proportional to their charges under the original $U(1)_I$ gauge groups, with proportionality constants d_I , and are always of roughly the same order as the original soft scalar $(\text{mass})^2$ (i.e. m_{SUSY}^2), even though the scale of spontaneous symmetry breaking set by (2.3) is many orders of magnitude larger [5].

In a general situation, the rank reduction process exemplified above may be iterated several times; then it is clear that the total D-term contributions to the surviving scalar fields from the spontaneously broken $U(1)_I$ are of the form

$$\Delta m_a^2 = \sum_I Q_{Ia} D_I \quad (2.6)$$

where now the parameters D_I reflect the more complicated features of the symmetry breaking process, and in general parameterize our ignorance of such details as which fields are getting VEVs and possible hierarchies in the VEVs.

In addition, if the spontaneously broken gauge groups included non-abelian generators T^α which commute with all of the generators of $SU(3)_C \times SU(2)_L \times U(1)_Y$, these could contribute residual soft mass terms of the form

$$\Delta L_{\text{soft}} = -D_\alpha (\varphi^\dagger T^\alpha \varphi) + \text{H.c.} \quad (2.7)$$

However, in the MSSM we may dismiss this possibility if the underlying gauge group is a subgroup of E_6 , or more generally if it does not mix families. This is because the only distinct chiral superfields with the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers in the MSSM are L and H_d , so that (2.7) would have to be of the form

$$\Delta L_{\text{soft}} = -D_L L^\dagger H_d + \text{H.c.} \quad (2.8)$$

which would necessarily imply R-parity violation. So we need only consider $U(1)_I$ D-terms. [It might be interesting to consider D-terms that could arise in variants of the MSSM with R-parity violation, or with family-dependent gauge symmetries (see e.g. [26]).]

The group E_6 contains two $U(1)$ factors not contained in the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. One may choose a basis for the two extra $U(1)$ factors in an arbitrary way, but it is important to realize that the original $U(1)_I$ groups (with canonical kinetic terms) above the symmetry-breaking scale will in general be linear combinations of the two chosen $U(1)$ basis groups and the surviving weak hypercharge $U(1)_Y$. We choose as a basis $U(1)_Y$, $U(1)_X$, and $U(1)_S$ as in Table 1.

Table 1: $U(1)$ charges of MSSM chiral superfields

	Q	u	d	L	e	H_u	H_d
$U(1)_Y$	1/6	-2/3	1/3	-1/2	1	1/2	-1/2
$U(1)_X$	-1/3	-1/3	1	1	-1/3	2/3	-2/3
$U(1)_S$	-1/3	-1/3	-2/3	-2/3	-1/3	2/3	1

One of the reasons behind this choice is that within a **27** of E_6 there are two standard model singlet components ν and S , which carry $[U(1)_Y, U(1)_X, U(1)_S]$ charges $[0, -5/3, 0]$ and $[0, 0, -5/3]$ respectively. Thus $U(1)_X$ could be broken by a VEV for the scalar component of the field ν (which carries lepton number -1), and $U(1)_S$ could be broken by the field S (which carries lepton number 0). The normalizations are chosen so that the largest charge appearing in Table 1 is unity; in order to obtain the correct normalization for unification into E_6 (or any of its subgroups), these charges should be multiplied by $\sqrt{\frac{3}{5}}, \frac{3}{2\sqrt{10}}, \frac{3}{2\sqrt{10}}$ respectively. The gauged $B - L$ subgroup of E_6 is given by

$$U(1)_{B-L} = \frac{4}{5}U(1)_Y - \frac{3}{5}U(1)_X.$$

There is also a non-abelian $SU(2)$ factor within E_6 which commutes with all of the surviving $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge generators. It generates simultaneous rotations $L \leftrightarrow H_d$, $\nu \leftrightarrow S$, and $d \leftrightarrow h$, where h is a non-MSSM color anti-triplet. As we mentioned above, the off-diagonal generators of this $SU(2)$ are prohibited by R-parity conservation from contributing to soft-scalar masses via D-terms. Its existence means, however, that there is a potential ambiguity in assigning $U(1)$ charges to H_d , L , and d , stemming from different embeddings of these fields into remnants of a **27** for certain subgroups of E_6 . The $L \leftrightarrow H_d$ ambiguity is easily fixed just by the choice of $U(1)_X$ versus $U(1)_S$. [Note that under $U(1)_X \leftrightarrow U(1)_S$, the charges of L and H_d are exchanged and those of (Q, u, e, H_u) remain invariant.] The $U(1)$ charges of d are then also uniquely fixed if we demand that the MSSM Yukawa couplings are allowed by the unbroken gauge group. Other assignments of the $U(1)_{X,S}$ charges for d would imply that $B - L$ cannot be a gauged $U(1)$ symmetry, a not particularly attractive possibility which we decline to pursue here.

The D-term contributions to the soft scalar masses of the MSSM arising from spontaneous breakdown of any subgroup of E_6 may therefore be parameterized by:

$$\begin{aligned}
\Delta m_{\tilde{Q}}^2 &= \frac{1}{6}D_Y - \frac{1}{3}D_X - \frac{1}{3}D_S; & \Delta m_{\tilde{e}}^2 &= D_Y - \frac{1}{3}D_X - \frac{1}{3}D_S; \\
\Delta m_{\tilde{u}}^2 &= -\frac{2}{3}D_Y - \frac{1}{3}D_X - \frac{1}{3}D_S; & \Delta m_{H_u}^2 &= \frac{1}{2}D_Y + \frac{2}{3}D_X + \frac{2}{3}D_S; \\
\Delta m_{\tilde{d}}^2 &= \frac{1}{3}D_Y + D_X - \frac{2}{3}D_S; & \Delta m_{H_d}^2 &= -\frac{1}{2}D_Y - \frac{2}{3}D_X + D_S. \\
\Delta m_{\tilde{L}}^2 &= -\frac{1}{2}D_Y + D_X - \frac{2}{3}D_S; & &
\end{aligned} \tag{2.9}$$

These corrections to the scalar masses should be applied at the scale M_D of spontaneous symmetry breaking. In order to see how these contributions affect physics at ordinary energies, we must use the RG equations. For the squarks and sleptons of the third family and the Higgs scalars, these are given by

$$\begin{aligned}
16\pi^2 \frac{dm_{\tilde{Q}}^2}{dt} &= -\frac{32}{3}g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15}g_1^2 M_1^2 + \frac{1}{5}g_1^2 S + 2y_t^2 \Sigma_t^2 + 2y_b^2 \Sigma_b^2 \\
16\pi^2 \frac{dm_{\tilde{u}}^2}{dt} &= -\frac{32}{3}g_3^2 M_3^2 - \frac{32}{15}g_1^2 M_1^2 - \frac{4}{5}g_1^2 S + 4y_t^2 \Sigma_t^2 \\
16\pi^2 \frac{dm_{\tilde{d}}^2}{dt} &= -\frac{32}{3}g_3^2 M_3^2 - \frac{8}{15}g_1^2 M_1^2 + \frac{2}{5}g_1^2 S + 4y_b^2 \Sigma_b^2 \\
16\pi^2 \frac{dm_{\tilde{L}}^2}{dt} &= -6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2 - \frac{3}{5}g_1^2 S + 2y_\tau^2 \Sigma_\tau^2 \\
16\pi^2 \frac{dm_{\tilde{e}}^2}{dt} &= -\frac{24}{5}g_1^2 M_1^2 + \frac{6}{5}g_1^2 S + 4y_\tau^2 \Sigma_\tau^2 \\
16\pi^2 \frac{dm_{H_u}^2}{dt} &= -6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2 + \frac{3}{5}g_1^2 S + 6y_t^2 \Sigma_t^2 \\
16\pi^2 \frac{dm_{H_d}^2}{dt} &= -6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2 - \frac{3}{5}g_1^2 S + 6y_b^2 \Sigma_b^2 + 2y_\tau^2 \Sigma_\tau^2
\end{aligned}$$

where $t \equiv \ln(Q/Q_0)$; $M_{3,2,1}$ are the running gaugino masses; $y_{t,b,\tau}$ are the running Yukawa couplings of the third family; $A_{t,b,\tau}$ are the corresponding scalar trilinear couplings; $\Sigma_t^2 \equiv m_{H_u}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2$; $\Sigma_b^2 \equiv m_{H_d}^2 + m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2 + A_b^2$; $\Sigma_\tau^2 \equiv m_{H_d}^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 + A_\tau^2$; and

$$S \equiv \text{Tr}(Ym^2) = m_{H_u}^2 - m_{H_d}^2 + \sum_{\text{families}} (m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2 + m_{\tilde{d}}^2 - m_{\tilde{L}}^2 + m_{\tilde{e}}^2).$$

The $U(1)_Y$ gauge couplings g_1 and α_1 are taken to be in a GUT normalization throughout this paper. The RG equations of the first and second family squarks and sleptons are the same except that the terms proportional to Yukawa couplings do not appear.

Now the quantities Σ_t^2 , Σ_b^2 and Σ_τ^2 are unaffected when (2.9) are applied, and this persists as the scalar masses evolve according to the RG equations. So the only changes in the running of the soft scalar masses occur because of the presence of the terms proportional to S . One finds

$$\frac{dS}{dt} = \frac{66}{5} \frac{\alpha_1}{4\pi} S$$

which has the solution

$$S(t) = S(t_0) \alpha_1(t) / \alpha_1(t_0) .$$

At the scale M_D associated with spontaneous symmetry breaking, we have from (2.9)

$$S(t_D) = 11D_Y + \frac{4}{3}D_X - \frac{1}{3}D_S$$

Then at any scale lower than t_D we find that the change induced in the soft scalar masses by virtue of (2.9) (compared to a template model in which the D-terms are not present and all other parameters are the same) is given by

$$\Delta m_a^2(t) = Y_a \left[r D_Y - \frac{4(1-r)}{33} D_X + \frac{(1-r)}{33} D_S \right] + Q_{Xa} D_X + Q_{Sa} D_S \quad (2.10)$$

with $r(t) \equiv \alpha_1(t) / \alpha_1(t_D)$ and Y_a , Q_{Xa} , Q_{Sa} the charges of the scalar φ_a under $U(1)_Y$, $U(1)_X$, $U(1)_S$ respectively. In general r decreases as one moves to lower scales; for M_D near the GUT scale, $r \approx .43$ near the electroweak scale. So, to a good approximation, the effect of RG running of the scalar masses is simply to reduce the D_Y contribution by a factor of r , while leaving the D_X and D_S contributions untouched. If one is treating D_Y , D_X , and D_S as parameters of our ignorance regarding the rank reduction mechanism, one may as well impose them at e.g. the GUT scale or at the electroweak scale. The error incurred in doing so can always be simply absorbed into a redefinition of D_Y . We will henceforth refer to the effective value of D_Y at the electroweak scale as \hat{D}_Y ; it is equal to the quantity in square brackets in (2.10).

In some special cases, it is possible to make statements about the relative sizes of D_Y , D_X , D_S . For example, in an $SO(10)$ SUSY GUT, in the limit that $SO(10)$ breaks to $SU(3)_C \times SU(2)_L \times U(1)_Y$ immediately at the GUT scale, one has

$$D_Y = D_S = 0; \quad D_X \neq 0 .$$

In the case that $SO(10)$ breaks down to a smaller group [e.g. $SU(4)_C \times SU(2)_L \times U(1)_R$] and only then to the MSSM far below the GUT scale, one has

$$D_S = 0; \quad D_Y, D_X \neq 0.$$

However, there is another possible embedding of the MSSM into a $SU(4)_C \times SU(2)_L \times U(1)$ subgroup of E_6 which does not fit within $SO(10)$; this would yield instead

$$D_X = 0; \quad D_Y, D_S \neq 0.$$

In the case that the gauge group is a rank 5 subgroup of E_6 obtained from a Calabi-Yau superstring compactification [27], one has

$$D_X = D_S \neq 0$$

(with D_Y possibly non-zero) because of the mandatory [28] presence of the subgroup $U(1)_{X'} = U(1)_X + U(1)_S$ in such models. It is useful to note that in general the magnitude of D_Y is expected to vanish in the limit that $U(1)_Y$ has a vanishing mixing angle with the spontaneously broken $U(1)$ subgroups, a limit which is often approximately realized when the scale of spontaneous symmetry breaking is not far below the GUT scale.

Experimental limits on sfermion masses imply constraints on D_Y , D_X , and D_S . The condition that the right-handed selectron mass be above the present experimental limit amounts to approximately

$$m_0^2 > (45 \text{ GeV})^2(1 - |\cos 2\beta|) - .15m_{1/2}^2 - \hat{D}_Y + \frac{1}{3}D_X + \frac{1}{3}D_S, \quad (2.11)$$

while the lower limit on the mass of the sneutrino from its contribution to the invisible width of the Z boson is approximately given by

$$m_0^2 > (41 \text{ GeV})^2(1 + 2.5|\cos 2\beta|) - .5m_{1/2}^2 + \frac{1}{2}\hat{D}_Y - D_X + \frac{2}{3}D_S. \quad (2.12)$$

(We assume $\tan \beta > 1$ throughout this paper, so $\cos 2\beta < 0$.) Either (2.11) or (2.12) typically sets the lower limit on m_0^2 for given values of D_Y , D_X , D_S .

Note that if $\widehat{D}_Y - \frac{1}{3}D_X - \frac{1}{3}D_S$ and $-\frac{1}{2}\widehat{D}_Y + D_X - \frac{2}{3}D_S$ are both large and positive, one may entertain the interesting and unusual possibility that m_0^2 is significantly negative. Negative running squared masses for squarks and sleptons at high scales do not necessarily imply that the squark and slepton fields acquire VEVs, as long as there are no exactly D-flat and F-flat directions involving only MSSM fields in the supersymmetric part of the scalar potential at high energies. The correct place to study the scalar potential is at the scale of the putative VEV, which is typically much smaller than M_U or M_{Planck} ; at this scale contributions to the running scalar (mass)² terms from gaugino loops will push them positive even if they were negative at very high scales, eliminating the purported VEV. The MSSM does have many directions which are exactly D-flat and F-flat at the renormalizable level. However, these can be lifted by non-renormalizable terms in the superpotential and by D-terms from additional gauge interactions which remain unbroken in the high energy theory. If for example $U(1)_S$ survives down to an intermediate scale $M_I \sim \sqrt{M_Z M_U}$ [as suggested by (2.3) with $n = 2$], then all D-flat directions above M_I would have to involve VEVs for H_u or H_d , since all other chiral superfields in the MSSM have $U(1)_S$ charges of the same (negative) sign. One can show that all such flat directions can be lifted in models with e.g. chiral superfields and superpotential interactions for gauge-singlet neutrinos. Therefore, to maintain maximal generality, positivity constraints on linear combinations of scalar (mass)² terms which correspond to D-flat and F-flat directions of the MSSM at the renormalizable level should be imposed only at M_I , which we take to be 10^{10} GeV. Such constraints are easily incorporated into computer programs which calculate the sparticle spectrum numerically, as described in the next section.

Even when the D-terms vanish, there is in the MSSM a possibility for m_0^2 at the unification scale (or at least its effective value as seen from low energies) to be very slightly negative, but only if $m_{1/2}$ is large enough. Numerically, we find that in models which satisfy all other constraints, $m_0^2 \geq 0$ for $m_{1/2} < 100$ GeV, and $m_0^2 > -(50 \text{ GeV})^2$ when $m_{1/2} < 400$ GeV if D-terms are negligible. Also note that a negative value of D_S pushes all squark and slepton masses higher while pushing the Higgs (mass)² parameters lower. For this reason, we find that D_S has essentially no lower bound from experimental constraints

on sfermion masses alone, but turns out to be constrained by Higgs mass bounds and the stability of the Higgs scalar potential.

Each of the contributions D_Y , D_X , D_S will split $m_{H_u}^2$ and $m_{H_d}^2$ (in addition to the splitting induced by the top Yukawa coupling) so that there is a potentially significant impact on electroweak symmetry breaking. In particular, the value of μ needed for correct symmetry breaking (for other parameters fixed) may be raised or lowered. To understand this, we may consider the tree-level relation

$$\mu^2 = \frac{1}{|\cos 2\beta|} (m_{H_d}^2 \cos^2 \beta - m_{H_u}^2 \sin^2 \beta) - \frac{1}{2} M_Z^2.$$

The change in μ^2 induced by the D-terms is therefore approximately

$$\Delta(\mu^2) = \frac{1}{|\cos 2\beta|} \left[-\frac{1}{2} \hat{D}_Y - \frac{2}{3} D_X + (\cos^2 \beta - \frac{2}{3} \sin^2 \beta) D_S \right]. \quad (2.13)$$

In models with universal boundary conditions and no D-terms, $|\mu|$ tends to scale with $\max(m_0, m_{1/2})$. If the quantity in square brackets in (2.13) is negative, $|\mu|$ can be substantially smaller than is otherwise allowed, with important implications for the neutralino and chargino masses and mixing angles and for the Higgs scalar sector.

The D-terms can also have a substantial impact on mass relations for the squarks and sleptons. For example, a sum rule given in [29] for the first and second family squarks and sleptons is modified to

$$2m_{\tilde{u}_R}^2 - m_{\tilde{d}_R}^2 - m_{\tilde{d}_L}^2 + m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2 + \frac{10}{3} \sin^2 \theta_W M_Z^2 |\cos 2\beta| = -\frac{10}{3} \hat{D}_Y \quad (2.14)$$

Note that D_X and D_S only affect this sum rule indirectly (and weakly) through \hat{D}_Y , while a large contribution to D_Y can cause a substantial deviation.

Another interesting sum rule is [29]

$$m_{\tilde{u}_R}^2 - m_{\tilde{d}_R}^2 = (.01) M_{\tilde{g}}^2 - |\cos 2\beta| (43 \text{ GeV})^2 - \hat{D}_Y + \frac{1}{3} D_S - \frac{4}{3} D_X. \quad (2.15)$$

The first two terms on the RHS of (2.15) are generally small and of opposite sign, so that $m_{\tilde{u}_R}$ and $m_{\tilde{d}_R}$ are always very close to equal in models without D-term or other

non-universality; this relationship can be modified in a dramatic way by the D-term contributions as we will see in the next section. Using this and similar tests, a substantial deviation from the $D_Y = D_X = D_S = 0$ condition might be discernible at an e^+e^- collider [30-32], yielding a strong (if not unambiguous) clue to physics at very high energies.

Of course, D-term contributions will generally not be the only non-universal imprints left on the soft scalar masses by physics at very high energies. The very presence of additional gauge groups at high energies means that there will be additional contributions to the RG running. In addition, there will be the effects of running the RG equations between M_U and M_{Planck} . In order to keep our exploration of the parameter space within reasonable limits, we will neglect such effects insofar as they cannot be absorbed into the ignorance parameters D_Y , D_X and D_S . This presumes that the additional gauge symmetry is broken not far below M_U and that all interactions above M_U are reasonably weak so that threshold and running effects are not overwhelming. This situation is preferred anyway if the apparent unification of gauge couplings is assumed to be not accidental.

It may be useful, however, to introduce a parameterization of the most general family-independent scalar mass non-universality at M_U . There is of course no unique way to pick such a parametrization, but we clearly want to arrange that the parameters D_Y , D_X , D_S appear also in the more general parameterization, that RG running can be formulated in as transparent a way as possible, and that the parameterization be invertible in terms of the running scalar squared masses at the scale where they are presumed to be family-independent. Since there are 7 independent masses ($m_{\tilde{Q}}^2, m_{\tilde{u}}^2, m_{\tilde{d}}^2, m_{\tilde{L}}^2, m_{\tilde{e}}^2, m_{H_u}^2, m_{H_d}^2$), there should be 6 parameters (in addition to the common m_0^2) which describe the non-universality at the input scale. We find it convenient to choose the contributions to scalar masses from the extra three parameters to be just proportional to those from the $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ gaugino loops to the RG equations for the scalar masses; we will call these contributions K_3 , K_2 and K_1 respectively. Thus one may parameterize the most general family-independent but non-universal scalar masses at the input scale (e.g. M_U or M_{Planck}

in a given theory) as follows:

$$\begin{pmatrix} m_{\tilde{Q}}^2 \\ m_{\tilde{u}}^2 \\ m_{\tilde{d}}^2 \\ m_{\tilde{L}}^2 \\ m_{\tilde{e}}^2 \\ m_{H_u}^2 \\ m_{H_d}^2 \end{pmatrix} = \begin{pmatrix} 1 & 1/6 & -1/3 & -1/3 & 1 & 1 & 1/36 \\ 1 & -2/3 & -1/3 & -1/3 & 1 & 0 & 4/9 \\ 1 & 1/3 & 1 & -2/3 & 1 & 0 & 1/9 \\ 1 & -1/2 & 1 & -2/3 & 0 & 1 & 1/4 \\ 1 & 1 & -1/3 & -1/3 & 0 & 0 & 1 \\ 1 & 1/2 & 2/3 & 2/3 & 0 & 1 & 1/4 \\ 1 & -1/2 & -2/3 & 1 & 0 & 1 & 1/4 \end{pmatrix} \begin{pmatrix} m_0^2 \\ D_Y \\ D_X \\ D_S \\ K_3 \\ K_2 \\ K_1 \end{pmatrix}. \quad (2.16)$$

This matrix is invertible, so that in principle, if one were given the scalar (mass)² parameters at the input scale, one could recover m_0^2 , $D_{Y,X,S}$ and $K_{3,2,1}$:

$$\begin{pmatrix} m_0^2 \\ D_Y \\ D_X \\ D_S \\ K_3 \\ K_2 \\ K_1 \end{pmatrix} = \begin{pmatrix} -1/2 & -2 & 5/2 & 1/6 & 1/2 & -7/3 & 8/3 \\ 3/10 & -3/5 & 3/10 & -3/10 & 3/10 & 0 & 0 \\ -3/10 & 3/5 & -3/10 & 1/10 & -3/10 & 1 & -4/5 \\ -3/10 & 3/5 & -3/10 & -1/2 & -3/10 & 1 & -1/5 \\ 1/2 & 5/3 & -7/6 & -1/2 & -1/2 & 5/3 & -5/3 \\ 3/4 & 3/4 & -3/2 & 1/4 & -1/4 & 5/4 & -5/4 \\ 0 & 3 & -3 & 0 & 0 & 3 & -3 \end{pmatrix} \begin{pmatrix} m_{\tilde{Q}}^2 \\ m_{\tilde{u}}^2 \\ m_{\tilde{d}}^2 \\ m_{\tilde{L}}^2 \\ m_{\tilde{e}}^2 \\ m_{H_u}^2 \\ m_{H_d}^2 \end{pmatrix}.$$

Of course, in practice it is difficult to imagine being able to reconstruct the soft masses at the input scale, especially $m_{H_u}^2$ and $m_{H_d}^2$. More importantly, the fact that the parameterization is invertible means that it is also general. This parameterization has several other nice features. First, the K_i contributions do not affect S , so that the running of the sfermion masses of the first two families are unaffected. Thus the imprint of non-zero K_i on the squark and slepton masses of the first two families is exactly the same at the TeV scale as at the input scale. In particular, the sum rules (2.14) and (2.15) are not affected by non-zero K_i s. At the low scale, the scalar masses of the first two families are given by an equation of the same form as (2.16), with D_X and D_S unchanged, and with the replacements $D_Y \rightarrow \hat{D}_Y$ and $K_i \rightarrow \hat{K}_i$, where \hat{D}_Y is just as before and

$$\begin{aligned} \hat{K}_i &= K_i + C_i(t); \\ C_i(t) &= \begin{pmatrix} 3/5 \\ 3/4 \\ 4/3 \end{pmatrix} \frac{2}{\pi} \int_t^{t_U} dt \alpha_i(t) M_i(t)^2, \end{aligned} \quad (2.17)$$

taking into account one-loop RG contributions and neglecting the small Yukawa couplings. Non-zero K_i s only enter the RG equations for the third family squarks and sleptons and the

Higgs scalars through the combinations Σ_t^2 , Σ_b^2 and Σ_τ^2 . These give additional contributions which may be described to a good approximation by the parameterization

$$\begin{aligned}\Delta m_{\tilde{t}_L, \tilde{b}_L}^2 &= -X_t - X_b; & \Delta m_{\tilde{t}_R}^2 &= -2X_t; & \Delta m_{\tilde{b}_R}^2 &= -2X_b; & \Delta m_{\tilde{\tau}_L, \tilde{\nu}_\tau}^2 &= -X_\tau; \\ \Delta m_{\tilde{\tau}_R}^2 &= -2X_\tau; & \Delta m_{H_u}^2 &= -3X_t; & \Delta m_{H_d}^2 &= -3X_b - X_\tau.\end{aligned}$$

When $\tan\beta$ is not large, X_b and X_τ are negligible.

In the MSSM, there is a large uncertainty in the numerical contribution of gluino loops to squark masses from the RG running, because the strong coupling constant is not very precisely known and because the function $C_3(t)$ in (2.17) runs quickly with scale below 1 TeV. Note that this uncertainty can effectively be absorbed into the non-universality parameter K_3 . Similarly, the additional RG effects from MSSM gaugino loops (but not heavy non-MSSM gaugino loops) between M_U and M_{Planck} , or above any intermediate scale in which the gauge couplings run differently (due to e.g. thresholds from heavy vector-like chiral superfields) can always be absorbed into the K_i . Other new physics at very high energies (e.g. gaugino loops for gauge generators which do not commute with $SU(3)_C \times SU(2)_L \times U(1)_Y$) may influence low energy physics through more complicated combinations of the parameters given above.

3. Phenomenological constraints and numerical results

In this section we will study some of the implications of D-term non-universality on SUSY phenomenology. To do so consistently requires that we simultaneously impose a variety of constraints from correct electroweak symmetry breaking, absence of color-breaking and charge-breaking global minima of the scalar potential, direct and indirect sparticle and Higgs boson mass limits [33], and a neutralino LSP. It is convenient to study the resulting constrained parameter space using a computer program which generates models randomly, imposing the constraints and calculating the resulting sparticle masses, mixing angles and couplings numerically. Here we describe the results of such an investigation, using a computer program similar to the one described in [22]. We first treat some special cases obtained by adding D-term contributions to a “template” model with universal

boundary conditions, to illustrate how experimental constraints limit the size of the D-terms and how the SUSY phenomenology is in turn modified by the D-terms. We will then turn to a much more general study of the parameter space allowed by arbitrary values of all parameters as constrained by experiment. In all cases, the D-terms are applied at a scale $M_U \approx 2 \times 10^{16}$ where the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings unify, and we retain the universal boundary conditions on gaugino masses and scalar trilinear couplings at M_U .

We choose as a template model the MSSM with the following parameters fixed:

$$\begin{aligned} m_0^2 &= (100 \text{ GeV})^2; & m_{1/2} &= A_0 = 100 \text{ GeV}; \\ m_{\text{top}} &= 175 \text{ GeV}; & \tan \beta &= 3; & \text{sign}(\mu) &= +; \\ D_Y &= D_X = D_S = 0. \end{aligned}$$

Now consider turning on the D-terms. As explained in section 2, the following (approximate) directions in parameter space may be theoretically preferred:

$$\begin{aligned} \text{(A)} \quad & D_X \neq 0; \quad D_Y = D_S = 0, \\ \text{(B)} \quad & D_S \neq 0; \quad D_Y = D_X = 0, \\ \text{(C)} \quad & D_S = D_X \neq 0; \quad D_Y = 0. \end{aligned}$$

Therefore, we will investigate as examples what happens in each of cases A, B, and C separately.

Case A: In Figure 1 we plot $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$, m_h and m_A as a function of D_X , with $D_Y = D_S = 0$ and all other parameters fixed as in the template model. (In our notation, h and A are the lightest Higgs scalar and pseudoscalar boson, respectively.) The lower bound on D_X in this example is set by the sneutrino contribution to the invisible width of the Z boson, which translates to a limit $m_{\tilde{e}_L} > 82 \text{ GeV}$ in this example because of the sum rule

$$m_{\tilde{e}_L}^2 - m_{\tilde{\nu}}^2 = |\cos 2\beta| M_W^2. \quad (3.1)$$

The upper bound on D_X is set by the experimental bound [33] $m_h > 44 \text{ GeV}$. In the remaining range $-(95 \text{ GeV})^2 < D_X < (160 \text{ GeV})^2$, all other sparticles have experimentally allowed masses. Throughout most of this allowed range, m_A is sufficiently large compared

to m_h so that h behaves essentially like a Standard Model Higgs boson. However, as D_X approaches its upper bound, m_A and m_h become small together, as does μ . This has potentially important implications for Higgs production via $e^+e^- \rightarrow Zh$, the rate for which is proportional to the quantity $\sin^2(\beta - \alpha)$. [In the limit $m_A \gg m_h$, one has $\sin^2(\beta - \alpha) \approx 1$; the heavier Higgs scalar eigenstates decouple.] For D_X very close to its upper bound, the lightest Higgs mass eigenstate h has couplings which are not like a Standard Model Higgs boson; for $D_X > (156 \text{ GeV})^2$, we find in this example $\sin^2(\beta - \alpha) < 0.6$. However, as long as $D_X < (135 \text{ GeV})^2$, one has $\sin^2(\beta - \alpha) > 0.9$ here.

Case B: In Figure 2 we show $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$, m_h and m_A this time as a function of D_S added to the template model. As can be clearly seen from (2.9), $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$ and all other squark and slepton masses grow monotonically with increasingly negative D_S . Thus the lower bound on D_S is typically set by the lightest Higgs mass bound; here we find $D_S > -(325 \text{ GeV})^2$. Conversely, the upper bound on D_S is usually set by the lower experimental bound on either $m_{\tilde{e}_R}$ or (as in this case) $m_{\tilde{\nu}}$. For D_S near its experimental lower bound, h behaves very differently from a Standard Model Higgs, with $\sin^2(\beta - \alpha) < 0.6$ for $D_S < -(310 \text{ GeV})^2$. However, as long as $D_S > -(260 \text{ GeV})^2$, we find $\sin^2(\beta - \alpha) > 0.9$ in this example.

Case C: In Figure 3, we again plot $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$, m_h and m_A , now as a function of $D_X = D_S$ with $D_Y = 0$ and all other parameters fixed as in the template model. In this case, nothing very remarkable happens to the Higgs masses or couplings over the entire allowed range for $D_X = D_S$. Both m_A and m_h fall somewhat as $D_X = D_S$ increases, but $\sin^2(\beta - \alpha) > 0.96$ always in this example. The lower bound on $D_X = D_S$ is set by the sneutrino contribution to the invisible width of the Z boson (or, equivalently, by $m_{\tilde{e}_L} > 82 \text{ GeV}$ for this value of $\tan \beta$). The upper bound on $D_X = D_S$ is set by the lower limit of 45 GeV on $m_{\tilde{e}_R}$. Note that for $D_X = D_S$ less than only $-(59 \text{ GeV})^2$, one finds $m_{\tilde{e}_R} > m_{\tilde{e}_L}$, in contradistinction to the usual situation in models with universal boundary conditions.

The rate for Higgs scalar boson production from $e^+e^- \rightarrow Zh$ is equal to $\sin^2(\beta - \alpha)$ times the corresponding rate for a Standard Model Higgs scalar boson of the same mass. The situation (illustrated by cases A and B) of light pseudoscalar masses together with

small values of $\sin^2(\beta - \alpha)$ can occur for all values of $\tan\beta$ and m_{top} when appropriate D-terms are present, unlike in the case of universal boundary conditions. For all values of $\tan\beta$, we found some models for which $\sin^2(\beta - \alpha)$ was very close to zero. This generally happens near the “edge” of D-term parameter space, i.e., for values of D_Y , D_X , and D_S near their limits. When this occurs, the production cross-section for h is reduced, but the rate for hA production is correspondingly enhanced, so that hA production may be observable at LEP2.

We now consider the possibility of arbitrary values for all of the parameters. The remaining results of this section were obtained by an exhaustive exploration of the parameter space, with the following constraints:

$$\begin{aligned}
& -(200 \text{ GeV})^2 < m_0^2 < (800 \text{ GeV})^2; \\
& 40 \text{ GeV} < m_{1/2} < 400 \text{ GeV}; \\
& 1.5 < \tan\beta < 60; \quad \text{sign}(\mu) = \pm; \\
& 160 \text{ GeV} < m_{\text{top}} < 190 \text{ GeV}; \\
& -(500 \text{ GeV})^2 < D_Y, D_X, D_S < (500 \text{ GeV})^2.
\end{aligned} \tag{3.2}$$

As explained in the previous section, we allow for the possibility of negative m_0^2 . (We have checked, however, that this possibility does not have a strong impact on the general results outlined below, although it clearly can affect physics within individual models.) We impose constraints on negative squark and slepton running (mass)² terms at $M_I = 10^{10}$ GeV corresponding to each of the flat directions (\tilde{u}, \tilde{e}) ; (\tilde{u}, \tilde{d}) ; (\tilde{Q}, \tilde{L}) ; (\tilde{d}, \tilde{L}) ; and (\tilde{e}, \tilde{L}) (in each case with appropriate flavor structure). Here (\tilde{u}, \tilde{e}) denotes for example the D-flat and F-flat direction

$$\tilde{u}_R = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \quad \tilde{c}_R = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \quad \tilde{t}_R = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \quad \tilde{\tau}_R = \sqrt{2}v,$$

which implies the constraint $2m_{\tilde{u}_R}^2 + m_{\tilde{t}_R}^2 + 2m_{\tilde{\tau}_R}^2 > 0$ at M_I .

The range allowed for A_0 was determined for each model by the requirement that there be no color-breaking global minima of the scalar potential at the electroweak scale in the D-flat (but not F-flat) directions $\langle \tilde{t}_L \rangle = \langle \tilde{t}_R \rangle = \langle H_u \rangle \neq 0$ and $\langle \tilde{b}_L \rangle = \langle \tilde{b}_R \rangle = \langle H_d \rangle \neq 0$.

In addition, we required that there not be global color-breaking minima in the non-D-flat directions $\langle \tilde{t}_L \rangle \neq 0$ or $\langle \tilde{t}_R \rangle \neq 0$, in each case with all other VEVs vanishing. The latter requirements amount to approximately

$$m_{\tilde{t}_L}^2 > -M_Z^2 |\cos(2\beta)| \left(\frac{\alpha_3 + 3\alpha_2/4}{3\alpha_2 + 9\alpha_1/5} \right)^{1/2} \quad (3.3)$$

$$m_{\tilde{t}_R}^2 > -M_Z^2 |\cos(2\beta)| \left(\frac{\alpha_3 + 4\alpha_1/5}{3\alpha_2 + 9\alpha_1/5} \right)^{1/2} \quad (3.4)$$

respectively at the weak scale. Here $m_{\tilde{t}_L}^2$ and $m_{\tilde{t}_R}^2$ are the running stop mass parameters in the absence of electroweak breaking, which appear in the usual stop mass matrix as

$$\begin{pmatrix} m_{\tilde{t}_L}^2 + m_{\text{top}}^2 - .35|\cos 2\beta|M_Z^2 & m_{\text{top}}(A_t + \mu \cot \beta) \\ m_{\text{top}}(A_t + \mu \cot \beta) & m_{\tilde{t}_R}^2 + m_{\text{top}}^2 - .15|\cos 2\beta|M_Z^2 \end{pmatrix}.$$

The constraints (3.3) and (3.4) effectively eliminate the possibility of a stop squark much lighter than m_{top} with a small stop mixing angle, and thus limit certain combinations of the D-terms. In this study we do not allow the K_i introduced in the previous section to vary because of the practical need to confine the scope of the investigations, although it would certainly be interesting to consider this. We should remark that the limits on the dimensionful parameters in (3.2) were motivated both by practical considerations (e.g. computer running time) and our theoretical prejudices. Some of the results below would have to be generalized if, for example, we allowed $m_0^2 < -(200 \text{ GeV})^2$, or larger D-terms. With that caveat, we now proceed to see how the allowed supersymmetric spectrum is modified by the presence of D-terms.

Let us first consider the fate of some correlations between sparticle masses which may be taken for granted in the case of universal boundary conditions.

In Figure 4, we show the allowed region in the $(m_{\tilde{e}_L}, m_{\tilde{e}_R})$ plane for all models with universal boundary conditions (bounded by solid lines), and the additional region (bounded by dashed lines) allowed for all models with arbitrary D-terms. In the universal case, viable models lie within a fairly restricted wedge; in particular, $m_{\tilde{e}_L} \geq m_{\tilde{e}_R}$ holds as a general rule. (If we relaxed the restriction $m_{1/2} < 400 \text{ GeV}$ in our original choice of parameter space, the region allowed by universal boundary conditions would be enlarged for $m_{\tilde{e}_L} > 280$

GeV.) With D-terms present, $m_{\tilde{e}_R}$ and $m_{\tilde{e}_L}$ can each independently essentially saturate their lower limits. The lower limit on $m_{\tilde{e}_L}$ in all models considered here is about 75 GeV; except for small $\tan\beta$, this limit comes from the sneutrino contribution to the invisible width of the Z and the sum rule (3.1). As we saw from our examples A, B, and C, the slepton masses often (but certainly not always) set the limits on the D-terms.

Figure 5 depicts in the same way the allowed regions in the $(m_{\tilde{e}_L}, m_{\tilde{u}_L})$ plane. Here the correlation in the universal boundary condition case is looser, because $m_{\tilde{u}_L}$ grows much faster with $m_{1/2}$ than $m_{\tilde{e}_L}$ does. In the presence of D-terms the allowed region is considerably larger, but not quite enough to saturate the experimental lower limits on squark masses when $m_{\tilde{e}_L} > 150$ GeV. Still, a substantial area is opened up for which $m_{\tilde{u}_L} < m_{\tilde{e}_L}$, a situation which does not occur in the case of universal boundary conditions. A similar inversion of the usual mass hierarchy can occur for each of the other pairs of squarks and sleptons, depending on the values of D_Y , D_X , and D_S .

The masses of the first and second family squarks are always very strongly correlated with each other in the case of universal boundary conditions. In Figure 6, we show the (extremely narrow) allowed region in the $(m_{\tilde{u}_R}, m_{\tilde{d}_R})$ plane with solid lines, and the much larger region allowed by D-term contributions is enclosed in the dashed lines. However, the correlation between the two squark masses is not completely destroyed by D-terms, in the sense that e.g. $m_{\tilde{u}_R} < 200$ GeV implies $m_{\tilde{d}_R} < 300$ GeV; the converse is not true, however. The squark masses do not always saturate their lower bounds from direct searches (although $m_{\tilde{d}_R}$ can be quite small), mainly because the relative magnitude of the D-terms is constrained by the lower limits on the slepton masses, by Higgs boson searches, and by correct electroweak symmetry breaking.

Figure 7 depicts the allowed regions in the $(m_{\tilde{u}_R}, m_{\tilde{d}_L})$ plane. The wedge (bounded by solid lines) allowed by universal boundary conditions is slightly wider than in Figure 6. This time the correlation between the squark masses is much stronger than in Figure 6, even for D-terms as large as allowed by other constraints. This can be easily understood from (2.9); since $m_{\tilde{u}_R}^2$ and $m_{\tilde{d}_L}^2$ obtain the same contribution from D_X and D_S , a big difference

between them can only occur if D_Y dominates. However, such a situation is limited by the lower bound on $m_{\tilde{e}_R}$ (if $D_Y > 0$) or by the sneutrino contribution to the invisible width of the Z boson (if $D_Y < 0$). There is an interesting region for which $m_{\tilde{u}_R} > m_{\tilde{d}_L}, m_{\tilde{u}_L}$; in this case the number of charginos and second-lightest neutralinos from gluino decays might be substantially increased over the usual expectation from universal boundary conditions. This can also occur if $m_{\tilde{d}_R} > m_{\tilde{d}_L}, m_{\tilde{u}_L}$.

The mass of \tilde{d}_R (and \tilde{b}_1) can even be much less than the lightest stop when D-terms are present. In Figure 8, we show the allowed regions in the $(m_{\tilde{d}_R}, m_{\tilde{t}_1})$ plane. It should also be noted that with D-terms present, \tilde{d}_R especially can be considerably lighter than the gluino, a sharp contrast from the usual rule in the case of universal boundary conditions that all first and second family squark masses are greater than about $.85 M_{\tilde{g}}$.

As remarked in section 2, large D-terms can also have a significant effect on the neutralino and chargino sectors, through their impact on the μ parameter as implied by (2.13). In much of the allowed parameter space in the MSSM with universal boundary conditions, μ is large compared to M_2 so that the LSP (\tilde{N}_1), the second-lightest neutralino (\tilde{N}_2), and the lightest chargino (\tilde{C}_1) are usually gaugino-like. In Figure 9, we show the allowed regions in the (μ, M_2) plane for universal scalar masses (within solid lines) and for non-zero D-terms (within dashed lines). In the latter case there is a substantial new region with relatively smaller μ . The region with $\mu \gg M_2$ is also enlarged. If we restricted ourselves to any particular fixed values of $\tan\beta$ and m_{top} , the enlargement of the allowed region in the (μ, M_2) plane would be even more dramatic. We find that eq. (2.13) is usually a quite good indicator of the effect of the D-terms on μ , even with full one-loop radiative corrections in the Higgs sector taken into account.

In the MSSM, there is a well-known correlation between the masses of the lightest chargino and the second-lightest neutralino. In the limit that $M_Z/(\mu \pm M_{1,2})$ is small, one finds[29]

$$m_{\tilde{C}_1} \approx m_{\tilde{N}_2} \approx M_2 - \frac{M_W^2(M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2}$$

However, when $|\mu| \approx M_2$, this expansion fails so that one might expect a rather different

relation between these masses. In Figure 10 we show the allowed regions in the $(m_{\tilde{C}_1}, m_{\tilde{N}_2})$ plane for universal and for D-term boundary conditions. The latter region is significantly larger, particularly for smaller μ, M_2 . On the other hand, we find that other important features of the chargino and neutralino sector are largely unchanged by the presence of D-terms. An important example concerns the mass differences $m_{\tilde{C}_1} - m_{\tilde{N}_1}$ and $m_{\tilde{N}_2} - m_{\tilde{N}_1}$ which are important in determining the visibility of a possible trilepton signal for SUSY at a hadron collider, or of chargino pair-production at an e^+e^- collider. We find that $m_{\tilde{C}_1} - m_{\tilde{N}_1} > 15$ GeV always within our parameter space (3.2), even when $|\mu|$ is lowered with respect to M_2 by D-terms. The allowed regions in the $(m_{\tilde{N}_1}, m_{\tilde{C}_1})$ plane are shown in Fig. 11 for universal (enclosed in solid lines) and D-term (enclosed in dashed lines) boundary conditions. The charged lepton coming from chargino decay $\tilde{C}_1 \rightarrow \tilde{N}_1 l^+ \nu$ should thus always be energetic enough to serve as a trigger, if the corresponding branching ratio is large enough. The mass difference $m_{\tilde{N}_2} - m_{\tilde{N}_1}$ is also always larger than 7 GeV (and in the vast majority of models we found it was larger than 15 GeV). In Figure 12 we show the allowed regions in the $(m_{\tilde{N}_1}, m_{\tilde{N}_2})$ plane. Of course, these results are really more of a statement about the form of the neutralino and chargino mass matrices (assuming gaugino mass unification) after all present experimental constraints have been imposed than it is about the consequences of D-terms, which only affect this sector indirectly through their effect on μ .

[In the case that $\mu \ll m_{1/2}$, one could imagine having almost degenerate \tilde{C}_1, \tilde{N}_2 , and \tilde{N}_1 with large higgsino content. While this is a logical possibility, it is impossible to achieve in practice without extreme fine-tuning of parameters including D_Y, D_X , and D_S . In particular, it did not occur in the search of parameter space described above. This situation would also imply that the relic density of LSP's would be far too small for them to constitute the cold dark matter.]

We conclude this section by mentioning the effect of D-terms on the Z -peak observable $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$. The Standard Model prediction for this quantity is lower by about 2σ than the experimental value $R_b = 0.2202 \pm .0020$ obtained at LEP[34]. Supersymmetric corrections to R_b are almost always tiny [35-38]. In particular, in the

MSSM with universal boundary conditions, R_b never gets closer than 1.5σ to the central experimental value. In the most general supersymmetric scenario, the prediction for R_b can only be reconciled with the 1σ experimental range if certain strong conditions are met. For $\tan\beta < 30$, these include light stops which are predominantly \tilde{t}_R , and light charginos with a significant higgsino content. One might imagine that these conditions could be attained in the presence of D-terms. Somewhat surprisingly, we find that this is not the case; an examination of viable models satisfying the constraints (3.2) finds that the prediction for R_b can only slightly exceed the values indicated in Figure 1 of ref. [36], and never enter the 1σ experimentally allowed range. This may be understood qualitatively from the previous results; when the superpartners are near their experimental lower bounds (as required for larger R_b), one finds that only relatively mild deviations from universal boundary conditions can occur. The extent to which the mass of the lightest stop squark can be lowered without having a large stop mixing angle is limited by the constraint (3.4) on color-breaking minima. On the other hand, a large stop mixing angle generally implies that $|\mu|$ is relatively large (so that the lighter chargino is not higgsino-like) or that A_t is large (which is problematic because of possible color breaking minima of the scalar potential in a D-flat direction). This implies that the experimental value of R_b must fall if minimal SUSY is correct with universal gaugino mass boundary conditions and only D-term contributions to scalar mass non-universality as imposed here.

4. Conclusion

In this paper, we have examined the possibility of D-term contributions to soft scalar masses in the MSSM which arise if the unbroken gauge group at very high energies is an arbitrary subgroup of E_6 . These contributions are parameterized by three quantities D_Y , D_X , and D_S . The effect of one-loop RG evolution on these parameters can always be absorbed into a redefinition of D_Y . The sizes of D_Y , D_X , and D_S (for a given set of other parameters) are limited by direct and indirect constraints on sparticles (especially sleptons) and Higgs scalar boson masses, as well as from correct symmetry breaking.

Of course, there is no guarantee that the D-terms will be large enough to make their presence felt in comparison to the universal contributions to scalar masses from m_0^2 and gaugino loops. Furthermore, it is clear that a future observation of apparent scalar mass non-universality at M_U may not be unambiguously ascribed to D-terms, without further theoretical input. Still, checks of relationships which hold in the MSSM between squark and slepton masses will provide an important clue to physics at very high energies. With experimental constraints imposed, we found that the impact of the D-terms might be most dramatic on squark and slepton mass relations [see, for example, eq. (2.15) and Fig. 7]. In several important respects, we found that deviations from the universal boundary conditions tend to be rather mild, especially when superpartners are relatively light. For example, we did not succeed in finding a viable model with non-zero D-terms for which R_b was within 1σ of the present experimental value. We did find models at all values of $\tan\beta$ and m_{top} for which $\sin^2(\beta - \alpha)$ was close to zero, in which case Higgs production through the usual Standard Model mechanism $e^+e^- \rightarrow Zh$ will be impossible to detect. However, this generally occurred only when the D-terms were quite close to their experimental bounds, except for large $\tan\beta$. If scalar mass non-universality proves to be a phenomenological necessity, D-term contributions will be an attractive theoretical candidate, since they maintain the natural suppression of flavor-changing neutral currents.

Acknowledgments: We are grateful to Manuel Drees, Gordon Kane, Riccardo Rattazzi, and James Wells for their help. This work was supported in part by the U.S. Department of Energy.

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Figure Captions

Figure 1: The masses of the lightest Higgs scalar boson h (lower solid line), pseudoscalar Higgs boson A (upper solid line), left-handed selectron \tilde{e}_L (dashed line), and right-handed selectron \tilde{e}_R (dot-dashed line), as a function of the parameter D_X added to the template model described in the text.

Figure 2: The same quantities are plotted as in Figure 1, but now as a function of D_S added to the template model described in the text.

Figure 3: The same quantities are plotted as in Figure 1, but now as a function of $D_X = D_S$ added to the template model described in the text.

Figure 4: The allowed regions in the $(m_{\tilde{e}_L}, m_{\tilde{e}_R})$ plane for universal boundary conditions (inside solid lines) and arbitrary D-term boundary conditions (inside dashed lines), for models satisfying (3.2).

Figure 5: The allowed regions in the $(m_{\tilde{e}_L}, m_{\tilde{u}_L})$ plane.

Figure 6: The allowed regions in the $(m_{\tilde{u}_R}, m_{\tilde{d}_R})$ plane.

Figure 7: The allowed regions in the $(m_{\tilde{u}_R}, m_{\tilde{d}_L})$ plane.

Figure 8: The allowed regions in the $(m_{\tilde{d}_R}, m_{\tilde{t}_1})$ plane.

Figure 9: The allowed regions in the (μ, M_2) plane.

Figure 10: The allowed regions in the $(m_{\tilde{C}_1}, m_{\tilde{N}_2})$ plane.

Figure 11: The allowed regions in the $(m_{\tilde{N}_1}, m_{\tilde{C}_1})$ plane.

Figure 12: The allowed regions in the $(m_{\tilde{N}_1}, m_{\tilde{N}_2})$ plane.

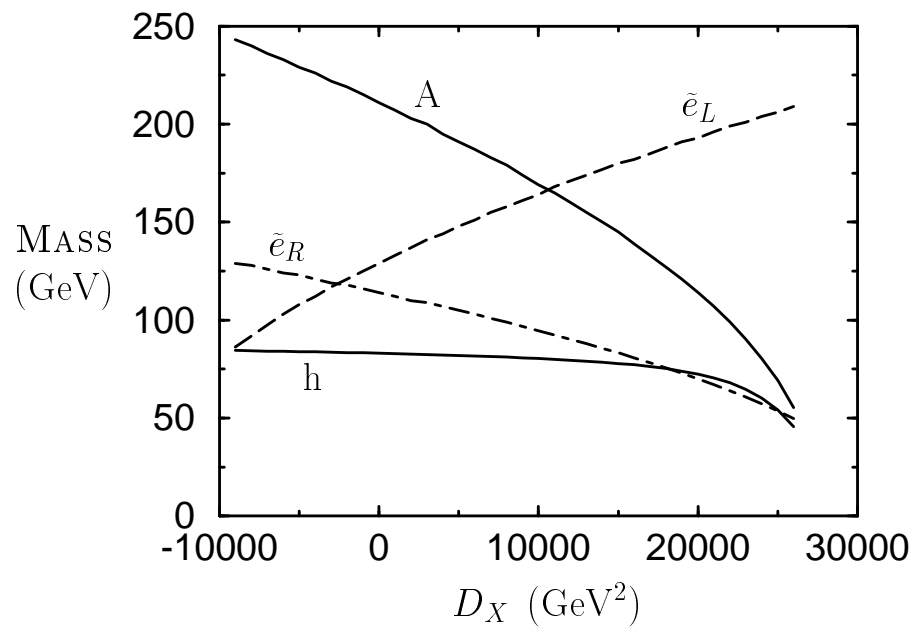


Figure 1

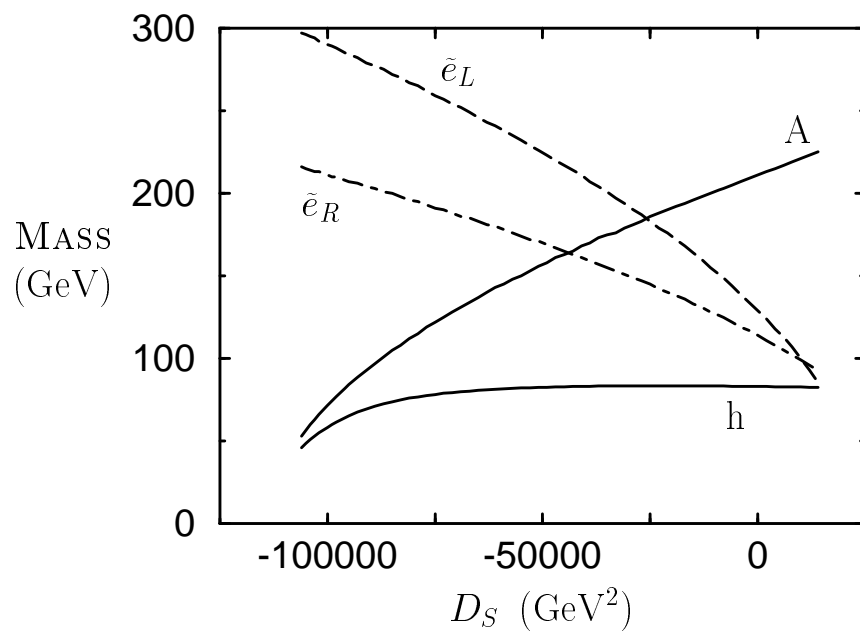


Figure 2

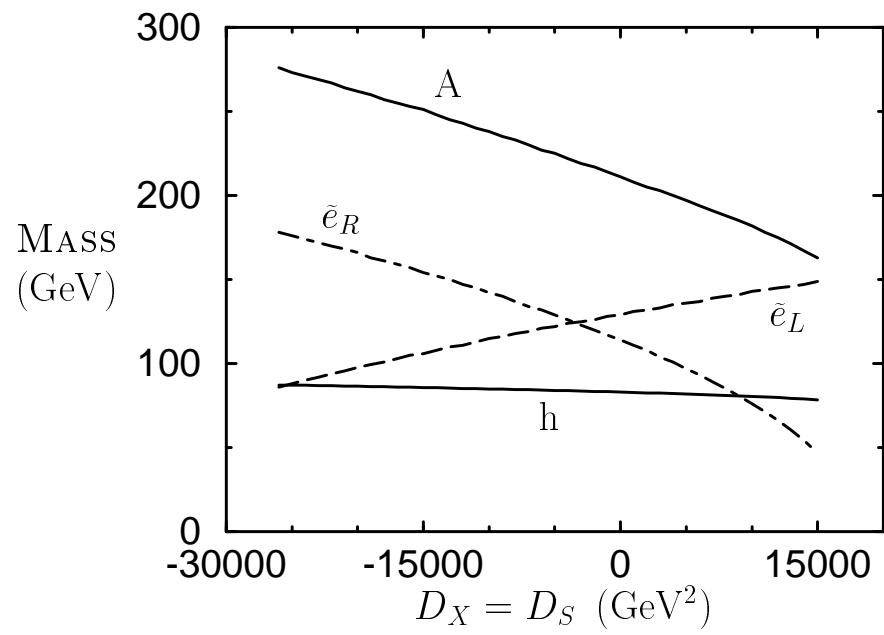


Figure 3

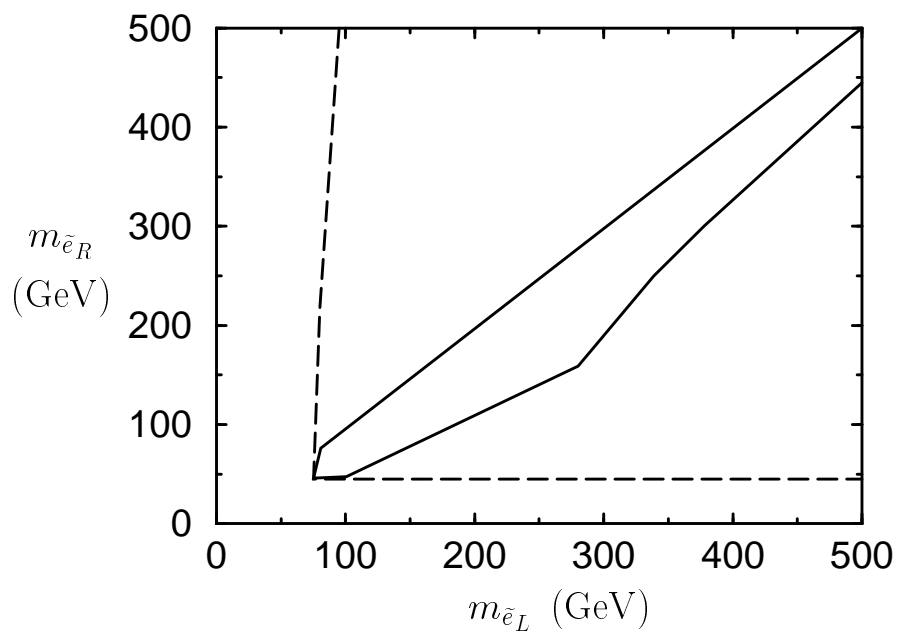


Figure 4

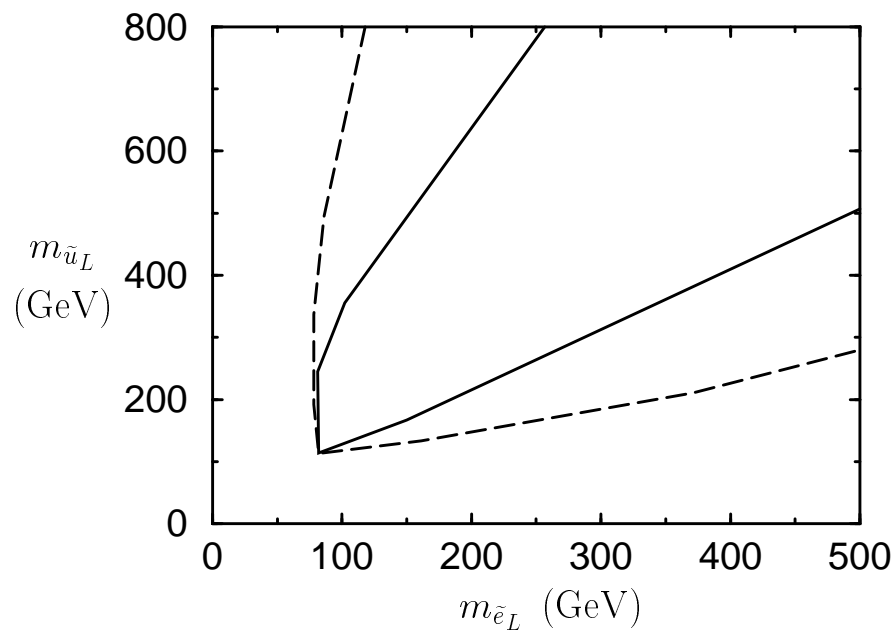


Figure 5

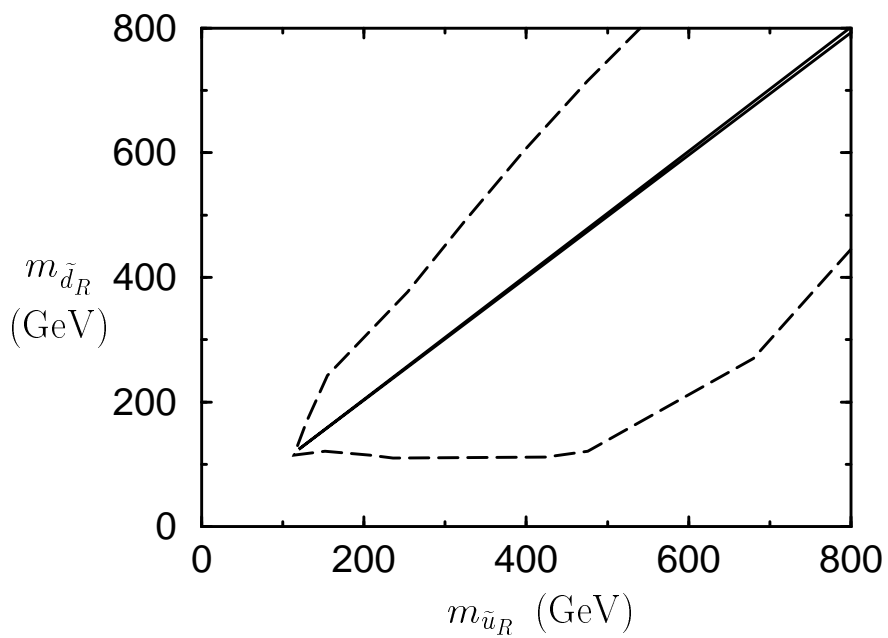


Figure 6

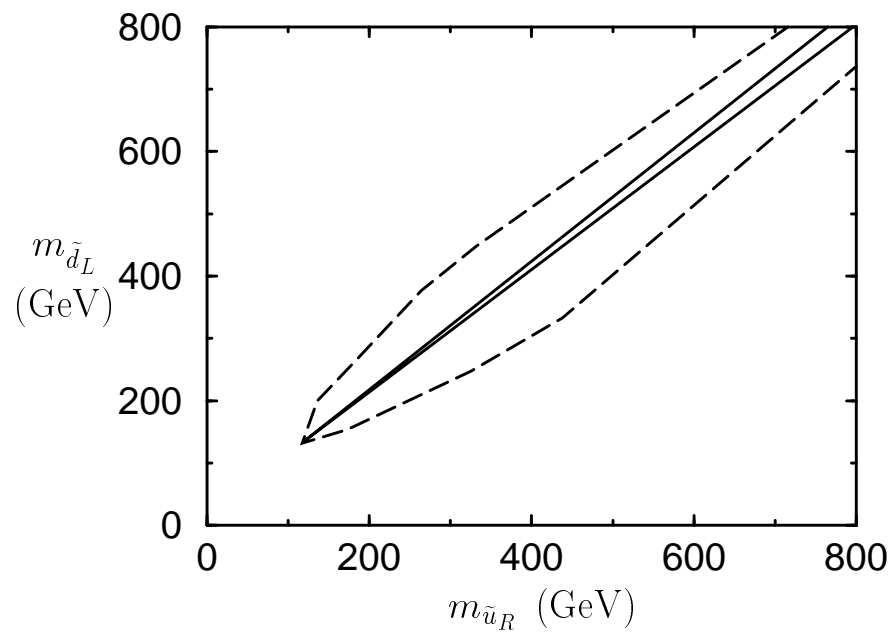


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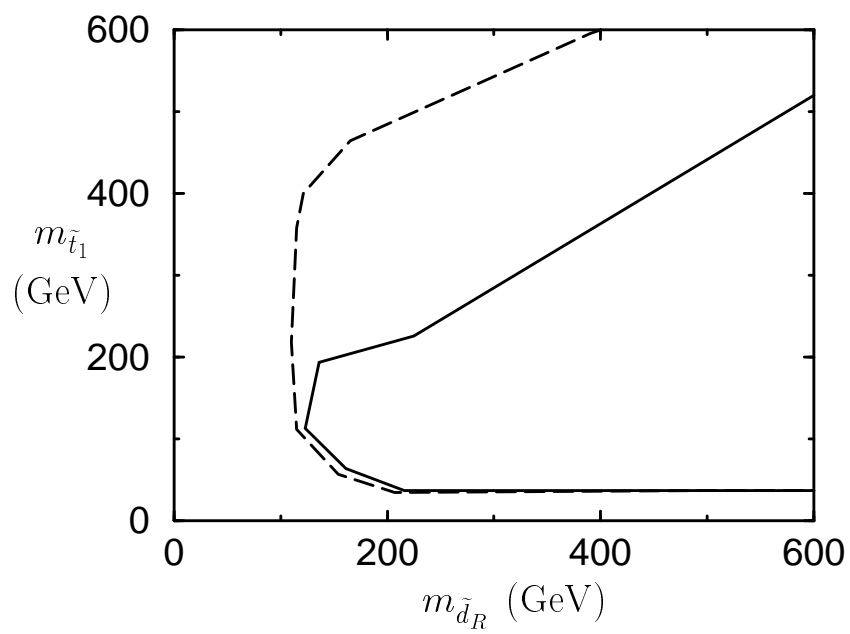


Figure 8

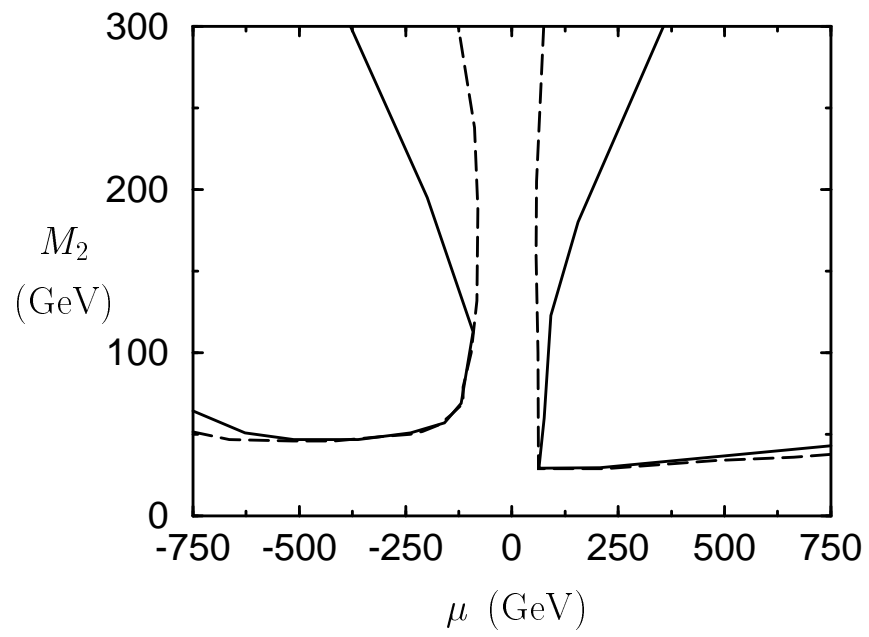


Figure 9

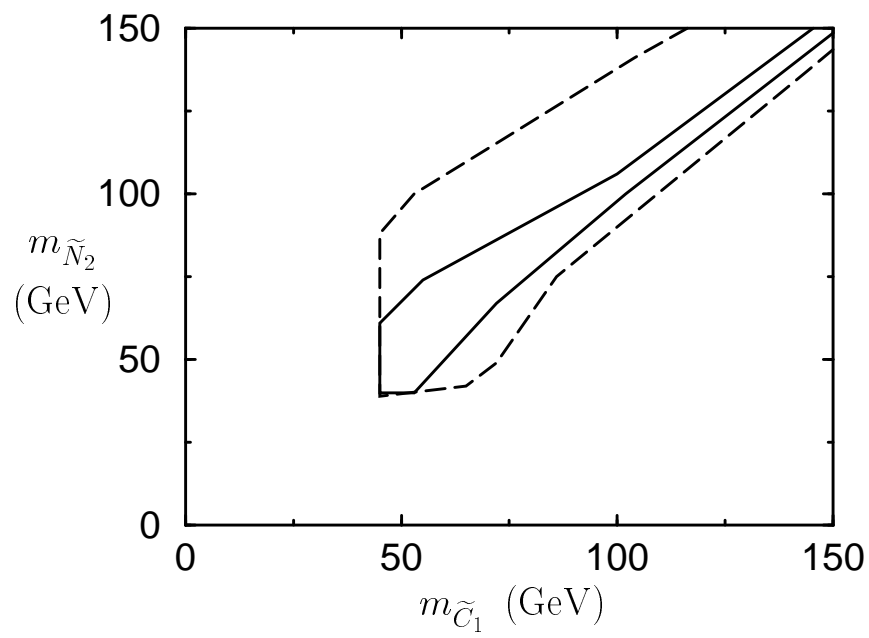


Figure 10

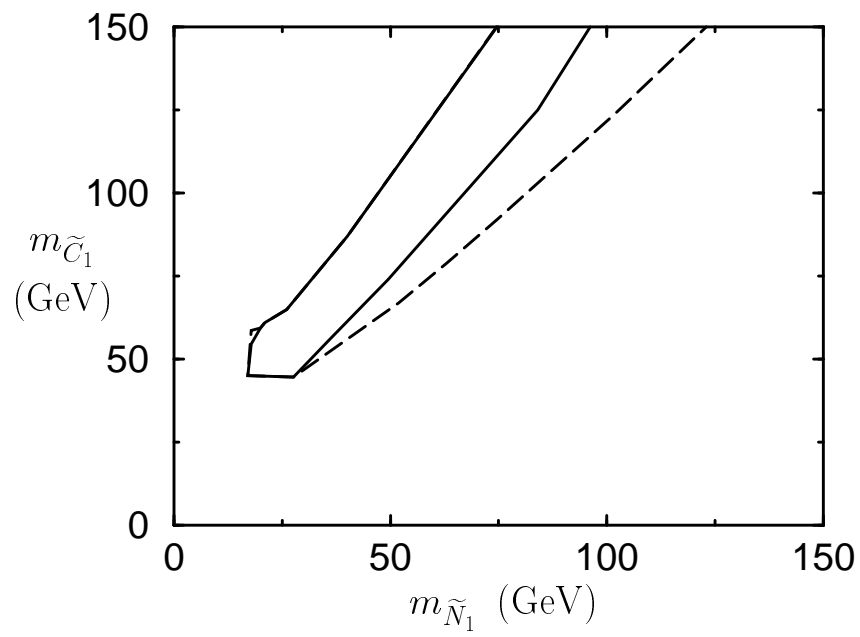


Figure 11

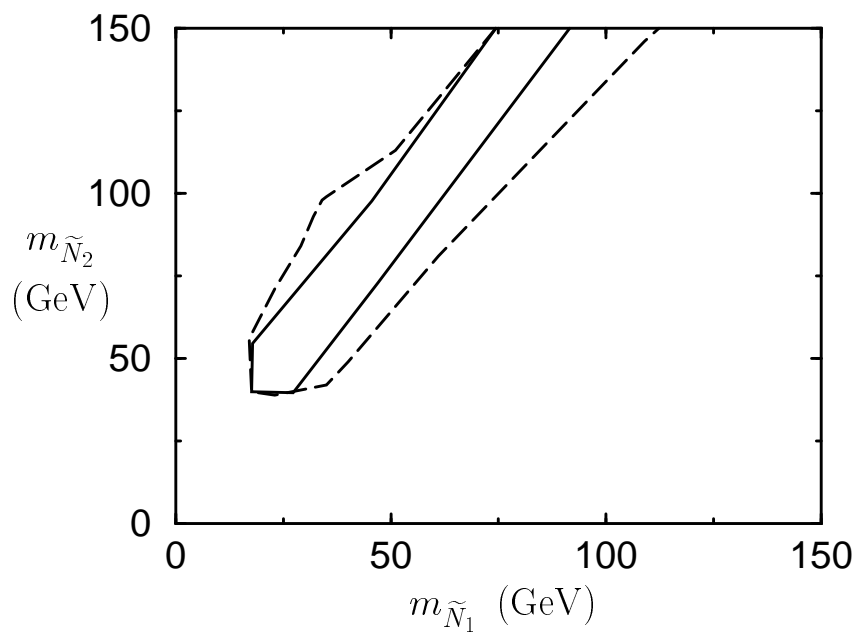


Figure 12